K04 FRAMES OF REFERENCE & RELATIVE VELOCITY

SPH4U

CH 1 (THE BIG PICTURE)

- the linear motion of objects in horizontal, vertical, and inclined planes
- the motion of a projectile in terms of components of its motion
- objects moving in two dimensions
- predict the motion of an object
- technological devices based on the concepts and principles of projectile motion

FRAMES OF REFERENCE

- **Frame of Reference:** coordinate system relative to which motion is observed

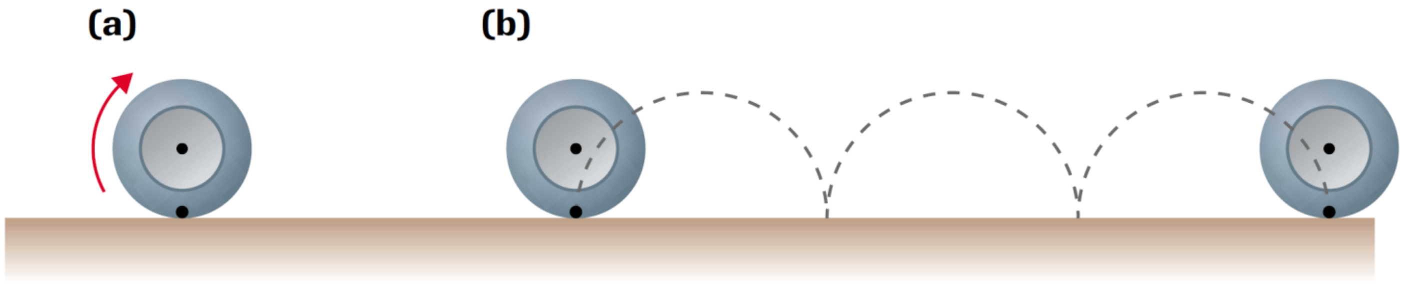

Figure 2
(a) The motion of a spot near the rim of a rolling wheel is simple if viewed from the frame of reference of the wheel's centre.
(b) The motion of the spot is much more complex when viewed from Earth's frame of reference.

FRAMES OF REFERENCE

- **Relative Velocity:** velocity of an object relative to a specific frame of reference

$$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$$

$$\vec{v}_{CE} = \vec{v}_{CW} + \vec{v}_{WE}$$

$$\vec{v}_{LO} = \vec{v}_{LM} + \vec{v}_{MN} + \vec{v}_{NO}$$

$$\vec{v}_{DG} = \vec{v}_{DE} + \vec{v}_{EF} + \vec{v}_{FG}$$

Figure 3
The pattern in relative velocity equations

SAMPLE PROBLEM 1

An Olympic canoeist, capable of travelling at a speed of 4.5 m/s in still water, is crossing a river that is flowing with a velocity of 3.2 m/s [E]. The river is 2.2×10^2 m wide.

(a) If the canoe is aimed northward, as in **Figure 4**, what is its velocity relative to the shore?

(b) How long does the crossing take?

(c) Where is the landing position of the canoe relative to its starting position?

(d) If the canoe landed directly across from the starting position, at what angle would the canoe have been aimed?

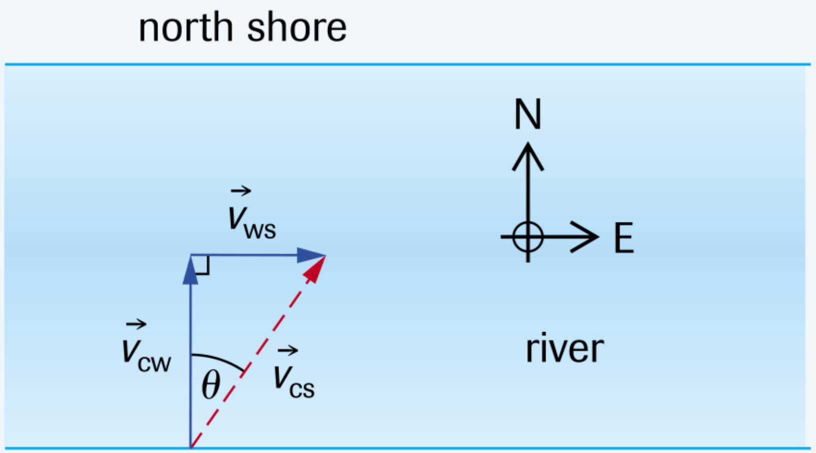

Figure 4
The situation

SAMPLE PROBLEM 1 – SOLUTIONS

Using the subscripts C for the canoe, S for the shore, and W for the water, the known relative velocities are:

$\vec{v}_{CW} = 4.5 \text{ m/s [N]}$

$\vec{v}_{WS} = 3.2 \text{ m/s [E]}$

(a) Since the unknown is $\vec{v}_{CS}$, we use the relative velocity equation

$$\vec{v}_{CS} = \vec{v}_{CW} + \vec{v}_{WS}$$

$$\vec{v}_{CS} = 4.5 \text{ m/s [N]} + 3.2 \text{ m/s [E]}$$

Applying the law of Pythagoras, we find:

$$|\vec{v}_{CS}| = \sqrt{(4.5 \text{ m/s})^2 + (3.2 \text{ m/s})^2}$$

$$|\vec{v}_{CS}| = 5.5 \text{ m/s}$$

Trigonometry gives the angle θ in **Figure 4**:

$$\theta = \tan^{-1} \frac{3.2 \text{ m/s}}{4.5 \text{ m/s}}$$

$$\theta = 35°$$

The velocity of the canoe relative to the shore is 5.5 m/s [35° E of N].

SAMPLE PROBLEM 1 – SOLUTIONS

(b) To determine the time taken to cross the river, we consider only the motion perpendicular to the river.

$$\Delta\vec{d} = 2.2 \times 10^2 \text{ m [N]}$$

$$\vec{v}_{CW} = 4.5 \text{ m/s [N]}$$

$$\Delta t = ?$$

From $\vec{v}_{CW} = \frac{\Delta\vec{d}}{\Delta t}$, we have:

$$\Delta t = \frac{\Delta\vec{d}}{\vec{v}_{CW}}$$

$$= \frac{2.2 \times 10^2 \text{ m [N]}}{4.5 \text{ m/s [N]}}$$

$$\Delta t = 49 \text{ s}$$

The crossing time is 49 s.

SAMPLE PROBLEM 1 – SOLUTIONS

(c) The current carries the canoe eastward (downstream) during the time it takes to cross the river. The downstream displacement is

$$\Delta\vec{d} = \vec{v}_{WS}\Delta t$$

$$= (3.2\text{ m/s [E]})(49\text{ s})$$

$$\Delta\vec{d} = 1.6 \times 10^2\text{ m [E]}$$

The landing position is 2.2×10^2 m [N] and 1.6×10^2 m [E] of the starting position. Using the law of Pythagoras and trigonometry, the resultant displacement is 2.7×10^2 m [36° E of N].

SAMPLE PROBLEM 1 – SOLUTIONS

(d) The velocity of the canoe relative to the water, $\vec{v}_{CW}$, which has a magnitude of 4.5 m/s, is the hypotenuse of the triangle in **Figure 5**. The resultant velocity $\vec{v}_{CS}$ must point directly north for the canoe to land directly north of the starting position.

The angle in the triangle is

$$\phi = \sin^{-1}\frac{|\vec{v}_{WS}|}{|\vec{v}_{CW}|}$$

$$= \sin^{-1}\frac{3.2\text{ m/s}}{4.5\text{ m/s}}$$

$$\phi = 45°$$

The required heading for the canoe is [45° W of N].

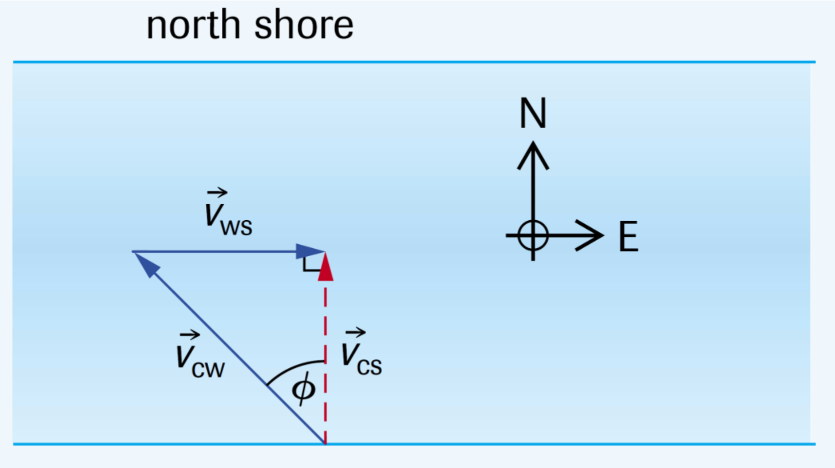

Figure 5
The solution for part (d)

SAMPLE PROBLEM 2

The air speed of a small plane is 215 km/h. The wind is blowing at 57 km/h from the west. Determine the velocity of the plane relative to the ground if the pilot keeps the plane aimed in the direction [34° E of N].

SAMPLE PROBLEM 2 – SOLUTIONS

We use the subscripts P for the plane, E for Earth or the ground, and A for the air.

$\vec{v}_{PA} = 215$ km/h [34° E of N]

$\vec{v}_{AE} = 57$ km/h [E]

$\vec{v}_{PE} = ?$

$$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$$

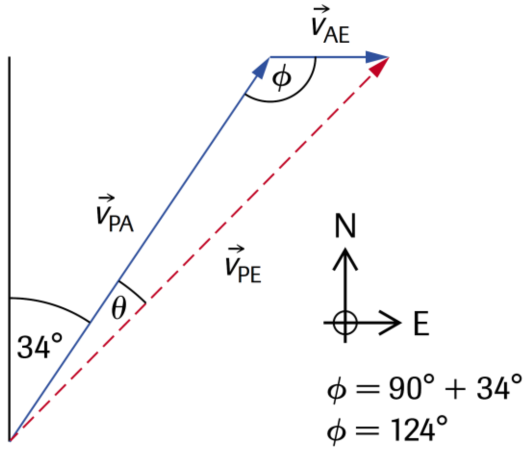

Figure 6
Solving Sample Problem 2 using trigonometry

This vector addition is shown in **Figure 6.** We will solve this problem by applying the cosine and sine laws; however, we could also apply a vector scale diagram or components as described in Appendix A.

SAMPLE PROBLEM 2 – SOLUTIONS

Using the cosine law:

$$\begin{aligned}
\left|\vec{v}_{PE}\right|^2 &= \left|\vec{v}_{PA}\right|^2 + \left|\vec{v}_{AE}\right|^2 - 2\left|\vec{v}_{PA}\right|\left|\vec{v}_{AE}\right| \cos \phi \\
&= (215\ \text{km/h})^2 + (57\ \text{km/h})^2 - 2(215\ \text{km/h})(57\ \text{km/h}) \cos 124^\circ \\
\left|\vec{v}_{PE}\right| &= 251\ \text{km/h}
\end{aligned}$$

Using the sine law:

$$\frac{\sin \theta}{\left|\vec{v}_{AE}\right|} = \frac{\sin \phi}{\left|\vec{v}_{PE}\right|}$$

$$\sin \theta = \frac{57\ \text{km/h}\ (\sin 124^\circ)}{251\ \text{km/h}}$$

$$\theta = 11^\circ$$

The direction of $\vec{v}_{PE}$ is $34^\circ + 11^\circ = 45^\circ$ E of N. Thus $\vec{v}_{PE} = 251\ \text{km/h}$ [45° E of N].

SAMPLE PROBLEM 3

A helicopter, flying where the average wind velocity is 38 km/h [25° N of E], needs to achieve a velocity of 91 km/h [17° W of N] relative to the ground to arrive at the destination on time, as shown in **Figure 7**. What is the necessary velocity relative to the air?

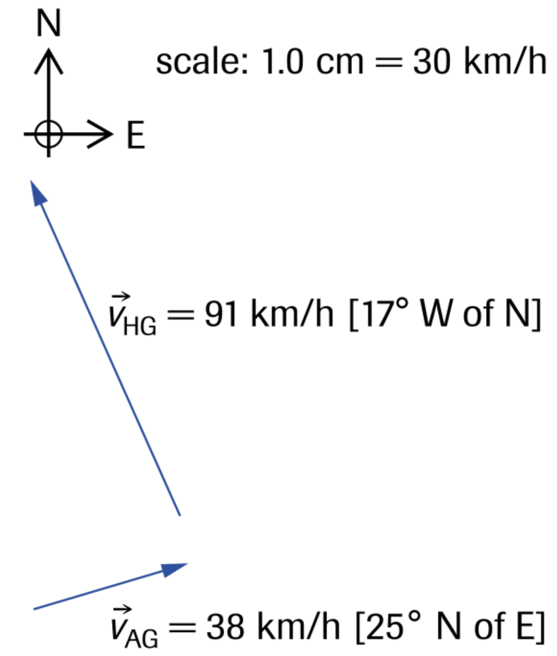

Figure 7
Situation for Sample Problem 3

SAMPLE PROBLEM 3 – SOLUTIONS

Using the subscripts H for the helicopter, G for the ground, and A for the air, we have the following relative velocities:

$$\vec{v}_{HG} = 91 \text{ km/h } [17° \text{ W of N}]$$
$$\vec{v}_{AG} = 38 \text{ km/h } [25° \text{ N of E}]$$
$$\vec{v}_{HA} = ?$$

$$\vec{v}_{HG} = \vec{v}_{HA} + \vec{v}_{AG}$$

We rearrange the equation to solve for the unknown:

$$\vec{v}_{HA} = \vec{v}_{HG} - \vec{v}_{AG}$$
$$\vec{v}_{HA} = \vec{v}_{HG} + (-\vec{v}_{AG}) \quad \text{where } -\vec{v}_{AG} \text{ is } 38 \text{ km/h } [25° \text{ S of W}]$$

Figure 8 shows this vector subtraction. By direct measurement on the scale diagram, we can see that the velocity of the helicopter relative to the air must be 94 km/h [41° W of N]. The same result can be obtained using components or the laws of sines and cosines.

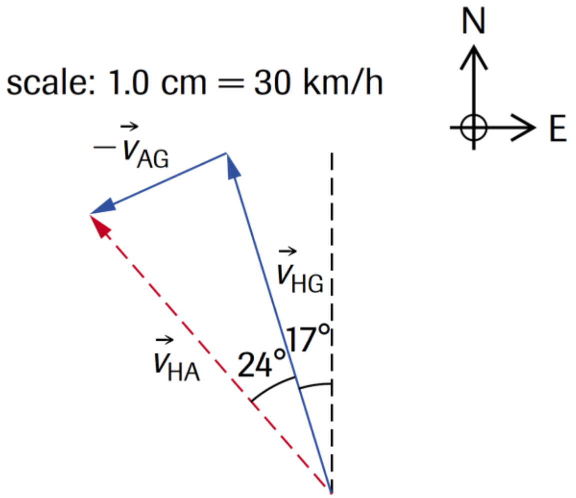

Figure 8
Solution to Sample Problem 3

SUMMARY

- A frame of reference is a coordinate system relative to which motion can be observed.
- Relative velocity is the velocity of an object relative to a specific frame of reference. (A typical relative velocity equation is $v\text{PE}$ $v\text{PA}$ $v\text{AE}$, where P is the observed object and E is the observer or frame of reference.)

PRACTICE

Readings

- Section 1.5 (pg 52)

Questions

- pg 57 #1-5